Appendix A: Model Foundations

A.1. DECENTRALIZED MODEL DERIVATIONS

A.1.1 Nonrenewable Sectors

We distinguish the nonrenewable sectors as mature sources of power generation, assuming they will not experience significant endogenous technological change relative to renewable sources. Each technology (*x*, *ng*, and *nu*) has production costs for source *i* $C_{it}(q_t^i)$ that are increasing and convex: $C_{it}'(q_t^i) > 0$ and $C_{it}''(q_t^i) > 0$. In our numerical model, we assume these supply curves are linear in the neighborhood of the price changes considered.

Profits for the representative firm of nonrenewable source *i* are revenues net of production costs and emissions and production taxes paid:

$$\pi^i = \nu_1\left((P_1 - f_1^i)q_1^i - C_{i1}(q_1^i) - \tau_1\mu^i q_1^i\right) + \delta\nu_2\left((P_2 - f_2^i)q_2^i - C_{i2}(q_2^i) - \tau_2\mu^i q_2^i\right)$$

The firm maximizes profits with respect to output from each fuel source, yielding the following first-order conditions:

$$\frac{\partial \pi^i}{\partial q_t^i} = 0: \quad P_t = C_{it}'(q_t^i) + f_t^i + \tau_t\mu^i. \qquad (.1)$$

Thus, each source of generation is used until its marginal costs—inclusive of their respective emissions costs—are equalized with each other and the price received.

A.1.2. Renewable Energy Sector

The renewable energy sector is both clean (nonemitting) and experiencing endogenous technological change. We divide this sector into a conventional technology (*w*), such as wind or biomass, and an advanced technology (*s*), like solar. We do include hydropower (*h2o*) in the baseline but assume it provides baseload capacity that does not change over time, in quantity or in cost.

To represent technological change, the costs of generation for renewable sources depend on a stock of knowledge that can be increased through research and development (R&D) or learning-by-doing (LBD). We assume that for $j = \{w, s\}$, these generation costs, $G_t(K_t^j, q_t^j)$, are increasing and convex in output and declining and convex its own knowledge stock, K_t^j, so that $G_q > 0$, $G_{qq} > 0$, $G_K < 0$, and $G_{KK} > 0$, where lettered subscripts denote derivatives with respect to the subscripted variable. Furthermore, since marginal costs are declining in knowledge and the cross-partials are symmetric, $G_{qK} = G_{Kq} < 0$.

The knowledge stock $K^j(H_t^j, Q_t^j)$ is a function of cumulative knowledge from R&D, H, and of cumulative experience through LBD, Q_t, where $K_H \geq 0$, $K_Q \geq 0$, and $K_{QH} = K_{HQ}$. Cumulative R&D-based knowledge increases in proportion to annual R&D knowledge generated in each stage, h_t, so $H_2 = H_1 + n_1 h_1$. Cumulative experience increases with total output during the first stage, so $Q_2 = Q_1 + n_1 q_1$. Research expenditures, $R^j(h_t^j)$, are increasing and convex in the amount of new R&D knowledge generated in any one year, with $R_h(h) > 0$ for $h > 0$, $R_h(0) = 0$, and $R_{hh} > 0$. The strictly positive marginal costs imply that real resources—specialized scarce inputs, employees, and equipment—must be expended to gain any new knowledge.[1] A subtle issue is whether research and experience are substitutes, in which case $K_{HQ} \leq 0$, or complements, making $K_{HQ} > 0$.

Two price-based policies are directly targeted at renewable energy: a renewable energy production subsidy (*s*), and a renewables technology R&D subsidy in which the government offsets a share (σ) of research expenditures.

In our two-stage model, profits for the representative nonemitting firm are

$$\pi^j = n_1\left((P_1 + s_1^j)q_1^j - G_1^j(K_1^j, q_1^j) - (1 - \sigma)R\left(h_1^j\right)\right) + \delta n_2\left((P_2 + s_2^j)q_2^j - G_2^j(K_2^j, q_2^j)\right),$$

where $K_2^j = K^j(H_2^j, Q_2^j)$.

Let ρ be a factor reflecting the degree of appropriability of returns from knowledge investments.[2] For example, $\rho = 1$ would reflect an extreme with perfect appropriability and no knowledge spillovers, while $\rho = 0$ reflects the opposite extreme of no private appropriability of knowledge investments. Similarly, $1 - \rho$ reflects the degree of knowledge spillovers. Importantly, these spillovers accrue as transfers within the sector, so the appropriation factor does not enter directly into the above representative profit function,

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[1] Because this is a partial equilibrium model, we do not explicitly explore issues of crowding-out in the general economy, but those opportunity costs may be reflected in the R&D cost function.

[2] We model general knowledge as being appropriable, with no distinction according to the source of that knowledge, whether R&D or learning. Although an empirical basis is lacking for such a distinction, one might expect that some forms of learning are less easily appropriated by other firms. We discuss the implication of relaxing this assumption in the context of the numerical simulations.

which reflects aggregate operating profits. However, the appropriation factor *does* enter into the first-order conditions for R&D and learning, since it determines the share of future profit changes that can be appropriated by the representative innovator.[3]

Differentiating the above profit function by the renewable firm's first-stage control variables (q_1, h_1), applying the appropriation factor to the benefits of innovation, and rearranging, we have (dropping the superscripts for now):

$$R_h(h_1) = - d\frac{r}{(1 - s)}n_2 G_K(K_2, q_2)K_H(H_2, Q_2) \qquad (.2)$$

$$G_q(K_1, q_1) = P_1 + s_1 - drn_2 G_K(K_2, q_2)K_Q(H_2, Q_2); \qquad (.3)$$

An important difference between the renewable and nonrenewable sectors is the response across time to policies. The nonrenewable sector's behavior depends only on current period prices and policies, but renewable sector responses are linked over time through innovation incentives. In the first stage, the firm invests in research until the discounted appropriated returns from additional R&D—lower production costs in the second stage—equal investment costs on the margin (equation A.2). By influencing future costs, policies in the second stage thus influence current private innovation decisions. Similarly, in equation (.3), each renewable energy source produces until the marginal cost of production equals the value it receives from additional output, including the market price, any production subsidy, and the appropriable contribution of such output to future cost reduction through learning-by-doing (note that the last term in equation (.3) is positive overall).

Second-stage output does not generate a learning benefit, so there is no related term in the first-order condition for q_2:

$$G_q(K_2, q_2) = P_2 + s_2. \qquad (.4)$$

Given the costs inherited from the knowledge investments in the first period, renewable energy providers simply equate the marginal costs with the net price received. Thus, for the same price effects, the renewable energy production decisions respond differently in the two periods.

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[3] This representation of aggregate appropriation as a share of the total benefits of innovation was formally derived in FN. We assume that all knowledge is ultimately adopted, either by imitation or by licensing. Licensing revenues do not appear in aggregate profits because they represent transfers among firms.

Note that if appropriation rates are imperfect $(\rho < 1)$, from a societal perspective, firms have insufficient incentive to engage in extra production for the purpose of learning-by-doing. Similarly, if the R&D subsidy does not fully reflect the spillover values $(\sigma < 1 - \rho)$, firms have insufficient incentive to invest in R&D. Thus, a knowledge externality accompanies the emissions externality, and both can be affected by policies that target one or the other.

A.1.3. Economic Surplus

We assume that when policies affect government revenues (which we denote as *V*), any changes in government revenues are compensated by (or returned in) lump-sum transfers. The amount of these transfers equals the tax revenues net of the cost of the subsidies:

$$\begin{aligned}\Delta V = {} & n_1\left(\sum_i \phi_1^i q_1^i + \tau_1 \sum_i \mu^i q_1^i - s_1^w q_1^w - s_1^s q_1^s - \sigma R(h_1) - b_{S1} Z_{S,1}(q_1^S) - b_L Z_L(q^L)\right) \\ & + \delta n_2\left(\sum_i \phi_2^i q_1^i + \tau_2 \sum_i \mu^i q_2^i - s_2^w q_2^w - s_2^s q_2^s - b_{S2} Z_{S,2}(q_2^S)\right)\end{aligned}$$

Environmental damages are a function of the annual emissions and the length of each stage; however, we will hold cumulative emissions constant across the policy scenarios, so a change in damages will not be a factor in the welfare comparisons. Our welfare measure is the change in *economic surplus* (excluding environmental benefits) due to a policy; it is then the sum of the changes in consumer and producer surplus and revenue transfers from the subsidy or tax: $\Delta W = \Delta U + \Delta \Pi + \Delta V$, where $\Delta \Pi = \sum_i \pi^i$.

Since consumer payments to firms and tax and subsidy payments are transfers, we can simplify the representation of economic surplus to be

$$\begin{aligned}W = {} & n_1\left(u(v_1) - Z_{S,1}(q_1^S) - Z_L(q^L) - \sum_{i=x,ng,nu} C_{i1}(q_1^i) - \sum_{j=w,s}\left(G^j(K_1^j, q_1^j) - R\left(h_1^j\right)\right)\right) \\ & + \delta n_2\left(u(v_2) - Z_{S,2}(q_2^S) - \sum_{i=x,ng,nu} C_{i2}(q_2^i) - \sum_{j=w,s}\left(G^j(K_2^j, q_2^j)\right)\right)\end{aligned}$$

A.2 SUPPLY FUNCTIONS AND PARAMETERIZATION

For nonrenewable sources of electricity generation, the cost functions all take the form $C_{it}(q_t^i) = c_{0t}^i + c_{1t}^i \times (q_t^i - q_{0t}^i) + c_{2t}^i \times (q_t^i - q_{0t}^i)^2 / 2$, where q_{0t}^i is the baseline (no policy) output in stage t for source i. Furthermore, from the first-order conditions for the baseline, the marginal cost of generation is $c_{1t}^i = P_{t,base}$. Total baseline cost, c_{0t}^i, does not affect nonrenewable energy decisions; we assume in effect zero profits in the baseline $(c_{0t}^i = P_{t,base} q_{t,base})$ to focus only on the changes in profits induced by policy.

The parameters for each generation source are calibrated to the EIA Annual Energy Outlook (AEO) 2013. Baseline generation levels (q_{it}^0) and emissions intensities (m^i) are likewise calibrated to NEMS model projections, namely the AEO 2013 Reference case. By comparing net prices and generation levels in the AEO side cases "No GHG Concern" and "GHG Policy Economy-wide," we derived the implicit slope (c_{2t}^i) parameters for each source in each time period.

We classify nonhydro renewables into two categories: solar (*s*) and wind/other conventional renewables (*w*) (wind, biomass, municipal solid waste, and geothermal) (IEA 2010a, 134). Their cost functions are inversely related to the knowledge stock, such that technological change lowers both the intercept and the slope of the renewables supply curve: $G_{jt}\left(K_t^j, q_t^j\right) = \left(g_{0t}^j + g_{1t}^j \times (q_t^j - q_{t,base}^j) + g_{2t}^j \times (q_t^j - q_{t,base}^j)^2 / 2\right)\left(K_{t,base}^j / K_t^j\right)$. The slope parameters (g_{2t}^j) are calibrated in the same manner as the other sources. The remaining renewables cost parameters (g_{1t}^j) are solved for in the baseline scenario, such that the first-order conditions hold. Since total baseline costs indicate the potential scope for cost reductions, we err on the high side (an optimistic assumption for optimal renewable generation subsidies) and normalize g_{0t}^j, such that baseline profits for renewable generation are zero. This parameter is varied in the sensitivity analysis (e.g., "Lowers incremental capacity costs").

From the first-order conditions, with these functional forms and that of the knowledge function, the baseline marginal cost is $g_{1,t}^j = P_{1,base} + k_1 drn_2 g_{0,2}^j / Q_{2,base}^j$.

A.3. CALIBRATION OF PARAMETERS

Table A1 reports the emissions intensities, knowledge function parameters, and demand parameters for the reference calibration. Unless otherwise specified, these remain the parameters for other scenarios.

Table A1

REFERENCE SCENARIO PARAMETERS

	BASE VALUE
CO_2 intensity of coal electricity (m^x) (tons CO_2/GWh)	983
CO_2 intensity of oil electricity (m^{oil}) (tons CO_2/GWh)	876
CO_2 intensity of natural gas electricity (m^{ng}) (tons CO_2/GWh)	401
Learning parameter for wind/other (k_1^w)	0.10
R&D parameter for wind/other (k_2^w)	0.15
Learning parameter for solar (k_1^s)	0.30
R&D parameter for solar (k_2^s)	0.20
Wind/other R&D cost parameter (g_0^w)	2.9×10^{10}
Wind/other R&D cost parameter (g_1^w)	1.2
Solar R&D cost parameter (g_0^s)	6.3×10^9
Solar R&D cost parameter (g_1^s)	1.2
Degree of knowledge appropriability (r)	0.5
Valuation rate for energy efficiency (b)	0.9
Very short run demand elasticity (e)	0.10
Short-run demand elasticity (h_{11})	0.20
Long-run demand elasticity (h_{22})	0.40
Cross-period demand elasticity (h_{12})	0.05

Table A2 reports the calibrated parameter values of the cost curves. The slopes of the supply curves can be interpreted as the increase in marginal cost in cents from an increase of 1 billion kWh in annual generation.

The marginal cost curves for energy efficiency investments are reported in dollars per percentage improvement in energy use per services. For example, from the baseline, an initial improvement in long-run energy efficiency requires an investment of \$1.1 trillion, and the marginal cost rises by \$3.4 trillion per percentage point.

Table A2.

SUPPLY AND DEMAND PARAMETERS BY STAGE[4]

	STAGE 1	STAGE 2
Slope of coal electricity supply (c_{2t}^{x}) (cent/billion kWh)	.00015	.001
Slope of natural gas electricity supply (c_{2t}^{ng}) (cent/billion kWh)	.004	.011
Slope of nuclear electricity supply, stage 2 (c_{2t}^{nu}) (cent/billion kWh)	—	.021
Slope of wind/other electricity supply (g_{2t}^{w}) (cent/billion kWh)	.021	.01
Slope of solar electricity supply (g_{2t}^{s}) (cent/billion kWh)	.17	.046
Intercept of short-run energy efficiency investment cost supply ($z_1^{S_1}$) (\$)	$.36 \times 10^{12}$	$.42 \times 10^{12}$
Slope of short-run energy efficiency investment cost supply ($z_2^{S_2}$) (\$/%)	7.7×10^{12}	1.2×10^{12}
Intercept of long-run energy efficiency investment cost supply (z_1^{L}) (\$)	1.1×10^{12}	—
Slope of long-run energy efficiency investment cost supply (z_2^{L}) (\$/%)	3.4×10^{12}	—
Exogenous demand growth	—	13%

A.4. DERIVATION OF ENERGY DEMAND PARAMETERS

To derive energy demand, we assume that the utility consumers derive from energy services is $u(v_t) = -A_t v_t^{-a}$, where A is a scalar that also allows for exogenous demand growth and $a > 0$. In period *t*, the quantity of energy demanded is $q_t = y_t v_t$, and we can equivalently write the consumer first-order condition for energy services as $aA_t \left(D_t / y_t\right)^{-a} / D_t = P_t$.

To be consistent with the notation used in FN, let us rewrite this expression in terms of the price elasticity of demand:

$$D_t = y_t^{\frac{a}{1+a}} \left(\frac{P_t}{aA_t}\right)^{\frac{-1}{1+a}} = N_t y_t^{1-e} P_t^{-e} \qquad (.5)$$

[4] The six parameters related to energy efficiency are derived given an assumption about the appropriation rate; these assume a base case where beta = 0.9.

where $a = (1 - e)/e$, $N_t = A_t^{e}(e/(1 - e))^{-e}$, and $0 < e < 1$.

The elasticity e can be interpreted as a very short run elasticity, as might be reflected in the rebound effect. Full short-run demand elasticity will include short-run responses in energy intensity. We derive these at the end.

Aggregate net consumer utility in the first stage of our two-stage model is then

$$
\begin{aligned}
U = {} & n_1\left(u(v_1) - P_1 v_1 y_1^0 e^{-(q_1^S + q^L)} - (1 - b_{S1})Z_{S,1}(q_1^S) - (1 - b_L)Z_L(q^L)\right) \\
& + d n_2\left(u(v_2) - P_2 v_2 y_2^0 e^{-(q_2^S + q^L)} - (1 - b_{S2})Z_{S,2}(q_2^S)\right)
\end{aligned}
$$

The representative consumer maximizes *perceived* net utility by choosing a level of energy services and rates of EE improvements in each stage (i.e., $v_1, v_2, q_1^S, q_2^S, q_1^L$). In period *t*, given any energy consumption rate per unit of service (which is determined simultaneously), the representative consumer maximizes utility with respect to *v*, resulting in the first-order condition

$$u'(v_t) = P_t y_t \qquad (.6)$$

Differentiating consumer utility with respect to short-run EE improvements, simplifying the expression for energy payments, and applying the EE valuation rate, we obtain the following first-order conditions in each stage:

$$(1 - b_{S2})Z'_{S,2}(q_2^S) = bP_2D_2 \qquad (.7)$$

$$(1 - b_{S1})Z'_{S,1}(q_1^S) = bP_1D_1 \qquad (.8)$$

In other words, consumers balance the marginal net cost of improving EE with the perceived energy costs of that period.

The choice of long-run EE improvements depends on both current and future energy spending, as well as the respective EE benefit valuation rates:

$$(1 - b_L)Z'_L(q^L) = bP_1D_1 + \frac{n_2}{n_1} bdP_2D_2 \qquad (.9)$$

Thus, policies that raise energy prices and thereby energy expenditures lead to increased investment in energy efficiency.

We assume linear marginal costs of EE improvements around the baseline, so for each type of improvement *j*, costs are a quadratic function $Z_j(q_t^j) = z_1^j q_t^j + z_2^j \times (q_t^j)^2 / 2$, with marginal costs $Z_j'(\theta_t^j) = \zeta_1^j + \zeta_2^j \cdot (\theta_t^j)$ and slope $Z_j''(\theta_t^j) = \zeta_2^j$. In the baseline $\theta_2^S = 0$, so from the first-order condition, we get $\zeta_1^S = \beta P_t^0 D_t^0$ and $\zeta_1^L = \beta P_1^0 D_1^0 + \frac{\nu_2}{\nu_1}\beta\delta P_2^0 D_2^0$.

Substituting these functional forms into the first-order conditions, we can derive the EE improvements:

$$\theta_2^S = \frac{\beta}{\zeta_2^{S_2}}\left(\frac{P_2 D_2}{(1-\beta_{S_2})} - P_2^0 D_2^0\right) \qquad (.10)$$

$$\theta_1^S = \frac{\beta}{\zeta_2^{S_1}}\left(\frac{P_1 D_1}{(1-\beta_{S_1})} - P_1^0 D_1^0\right) \qquad (.11)$$

$$\theta_1^L = \frac{\beta}{\zeta_2^{L}}\left(\frac{P_1 D_1}{(1-\beta_{L})} - P_1^0 D_1^0\right) + \frac{\nu_2}{\nu_1}\frac{\beta}{\zeta_2^{L}}\delta\left(\frac{P_2 D_2}{(1-\beta_{L})} - P_2^0 D_2^0\right) \qquad (.12)$$

The slopes of the marginal costs of EE improvements are thus important parameters, and we calibrate them by deriving the implicit short-, medium-, and long-run elasticities of electricity demand.

First, the elasticity of demand with respect to the energy intensity of services reflects the rebound effect, resulting from the very short run price elasticity ε:

$$\frac{\partial D_t}{\partial \psi_t} = (1-\varepsilon)N_t \psi_t^{-\varepsilon} P_t^{-\varepsilon}; \quad \frac{\partial D_t / D_t}{\partial \psi_t / \psi_t} = (1-\varepsilon)$$

The rebound effect recognizes that *v* will also change in response to lower costs of energy services, mitigating some of the energy savings. If *v* were unchanged, we would have an elasticity of 1.

The price elasticity of demand can be derived from the demand function. First,

$$\frac{dD_t}{dP_t} = -\varepsilon N_t \psi_t^{1-\varepsilon} P_t^{-\varepsilon-1} + (1-\varepsilon)N_t \psi_t^{-\varepsilon} P_t^{-\varepsilon}\left(\frac{\partial \psi_t}{\partial P_t} + \frac{\partial \psi_t}{\partial D_t}\frac{dD_t}{dP_t} + \frac{\partial \psi_t}{\partial D_s}\frac{dD_s}{dP_t}\right)$$

With $D_t / P_t = N_t \psi_t^{1-\varepsilon} P_t^{-\varepsilon-1}$, we can simplify

$$\frac{dD_t / D_t}{dP_t / P_t} = -\varepsilon + (1-\varepsilon)\left(\frac{\partial \psi_t / \psi_t}{\partial P_t / P_t} + \frac{\partial \psi_t / \psi_t}{\partial D_t / D_t}\frac{dD_t / D_t}{dP_t / P_t} + \frac{\partial \psi_t / \psi_t}{\partial D_s / D_s}\frac{dD_s / D_s}{dP_t / P_t}\right)$$

and solve for

$$\frac{dD_t / D_t}{dP_t / P_t} = \frac{-\varepsilon + (1-\varepsilon)\left(\frac{\partial \psi_t}{\partial P_t}\frac{P_t}{\psi_t} + \frac{\partial \psi_t}{\partial D_s}\frac{D_s}{\psi_t}\frac{dD_s}{dP_t}\frac{P_t}{D_s}\right)}{\left(1 - (1-\varepsilon)\frac{\partial \psi_t / \psi_t}{\partial D_t / D_t}\right)}$$

Thus, the elasticity is a combination of the very short run demand elasticity (absent changes in energy intensity) and the longer-run demand changes resulting from changes in energy intensity.

We also need to derive the "cross-price" elasticity of demand in one period with respect to the price in the other period. There is no direct effect on demand, but rather an indirect effect from changes in EE. Specifically, an increase in the other period's price increases long-run EE investments; however, some of these improvements will tend to be offset by fewer short-run investments.

$$\frac{dD_t}{dP_s} = (1-\varepsilon)\frac{D_t}{y_t}\left(\frac{\partial y_t}{\partial P_s} + \frac{\partial y_t}{\partial D_t}\frac{dD_t}{dP_s} + \frac{\partial y_t}{\partial D_s}\frac{dD_s}{dP_s}\right)$$

$$\Rightarrow \quad \frac{dD_t / D_t}{dP_s / P_s} = \frac{(1-\varepsilon)\left(\frac{\partial y_t}{\partial P_s}\frac{P_s}{y_t} + \frac{\partial y_t}{\partial D_s}\frac{D_s}{y_t}\frac{dD_s}{dP_s}\frac{P_s}{D_s}\right)}{\left(1-(1-\varepsilon)\frac{\partial y_t}{\partial D_t}\frac{D_t}{y_t}\right)}$$

Next, we derive the price elasticities of energy intensity:

$$\frac{\partial y_t}{\partial P_s} = -y_t\left(\frac{\partial q_t^S}{\partial P_s} + \frac{\partial q^L}{\partial P_s}\right) \quad \Rightarrow \quad \frac{\partial y_t / y_t}{\partial P_s / P_s} = -P_s\left(\frac{\partial q_t^S}{\partial P_s} + \frac{\partial q^L}{\partial P_s}\right)$$

$$\frac{\partial y_t}{\partial D_s} = -y_t\left(\frac{\partial q_t^S}{\partial D_s} + \frac{\partial q^L}{\partial D_s}\right) \quad \Rightarrow \quad \frac{\partial y_t / y_t}{\partial D_s / D_s} = -D_s\left(\frac{\partial q_t^S}{\partial D_s} + \frac{\partial q^L}{\partial D_s}\right)$$

From the simplified baseline first-order conditions (with no subsidies), we obtain the following partial derivatives:

$$\frac{\partial q_2^S}{\partial P_1}P_1 = \frac{\partial q_2^S}{\partial D_1}D_1 = 0; \quad \frac{\partial q_2^S}{\partial P_2}P_2 = \frac{\partial q_2^S}{\partial D_2}D_2 = \frac{\beta_2^S}{\zeta_2^{S_2}}P_2D_2;$$

$$\frac{\partial q_1^S}{\partial P_1}P_1 = \frac{\beta_1^S}{\zeta_2^{S_1}}D_1 = \frac{\partial q_1^S}{\partial q_1}P_1D_1; \quad \frac{\partial q_1^S}{\partial P_2}P_2 = \frac{\partial q_1^S}{\partial D_2}D_2 = 0;$$

$$\frac{\partial q_1^L}{\partial P_1}P_1 = \frac{\partial q_1^L}{\partial D_1}D_1 = \frac{\beta}{\zeta_2^L}P_1D_1; \quad \frac{\partial q_1^L}{\partial P_2}P_2 = \frac{\partial q_1^L}{\partial D_2}D_2 = \frac{\nu_2}{\nu_1}\frac{\beta}{\zeta_2^L}\delta P_2D_2;$$

which gives us

$$\frac{\partial y_1 / y_1}{\partial P_1 / P_1} = \frac{\partial y_1 / y_1}{\partial D_1 / D_1} = -\left(\frac{\beta}{\zeta_2^{S_1}} + \frac{\beta}{\zeta_2^L}\right)P_1D_1;$$

$$\frac{\partial y_2 / y_2}{\partial P_1 / P_1} = \frac{\partial y_2 / y_2}{\partial D_1 / D_1} = -\frac{\beta}{\zeta_2^L}P_1D_1;$$

$$\frac{\partial y_1 / y_1}{\partial P_2 / P_2} = \frac{\partial y_1 / y_1}{\partial D_2 / D_2} = -\delta\frac{\nu_2}{\nu_1}\frac{\beta}{\zeta_2^L}P_2D_2;$$

$$\frac{\partial y_2 / y_2}{\partial P_2 / P_2} = \frac{\partial y_2 / y_2}{\partial D_2 / D_2} = -\left(\frac{\beta}{\zeta_2^{S_2}} + \delta\frac{\nu_2}{\nu_1}\frac{\beta}{\zeta_2^L}\right)P_2D_2.$$

Let $h_{ts} \equiv -\dfrac{dD_t / D_t}{dP_s / P_s}$ be the (absolute value of) the price elasticity of demand. Thus, the own- and cross-price elasticities are

$$h_{11} = \frac{e + (1 - e)\left(\left(\frac{b}{z_2^{S_1}} + \frac{b}{z_2^{L}}\right)P_1 D_1 - d\frac{n_2}{n_1}\frac{b}{z_2^{L}}P_2 D_2 h_{21}\right)}{\left(1 + (1 - e)\left(\frac{b}{z_2^{S_1}} + \frac{b}{z_2^{L}}\right)P_1 D_1\right)};$$

$$h_{22} = \frac{e + (1 - e)\left(\left(\frac{b}{z_2^{S_2}} + \frac{n_2}{n_1}\frac{b}{z_2^{L}}d\right)P_2 D_2 - \frac{b}{z_2^{L}}P_1 D_1 h_{12}\right)}{\left(1 + (1 - e)\left(\frac{b}{z_2^{S_2}} + \frac{n_2}{n_1}\frac{b}{z_2^{L}}d\right)P_2 D_2\right)};$$

$$h_{12} = \frac{(1 - e)\frac{n_2}{n_1}\frac{b}{z_2^{L}}dPD_2\left(1 - h_{22}\right)}{\left(1 + (1 - e)\left(\frac{b}{z_2^{S_1}} + \frac{b}{z_2^{L}}\right)P_1 D_1\right)};$$

$$h_{21} = \frac{(1 - e)\frac{b}{z_2^{L}}P_1 D_1\left(1 - h_{11}\right)}{\left(1 + (1 - e)\left(\frac{b}{z_2^{S_2}} + \frac{n_2}{n_1}\frac{b}{z_2^{L}}d\right)P_2 D_2\right)}.$$

Setting these expressions to equal our target elasticities ($h_{11} = 0.2$, $h_{22} = 0.4$, and $h_{21} = 0.5$), we solve for our calibrated values of $z_2^{S_1}, z_2^{S_2}, z_2^{L}$:

$$z_2^{S_1} = bP_1^0 D_1^0 \frac{(1 - e)((1 - h_{11})(1 - h_{22}) - h_{12}h_{21})}{h_{11}(1 - h_{22}) - (1 - h_{12})h_{21} - e(1 - h_{21} - h_{22})};$$

$$z_2^{S_2} = bP_2^0 D_2^0 \frac{(1 - e)((1 - h_{11})(1 - h_{22}) - h_{12}h_{21})}{h_{22}(1 - h_{11}) - (1 - h_{21})h_{12} - e(1 - h_{11} - h_{12})};$$

$$z_2^{L} = d\frac{n_2}{n_1} bP_2^0 D_2^0 \frac{(1 - h_{11})(1 - h_{22}) - h_{12}h_{21}}{h_{12}};$$

and the relationship between the cross-price elasticities:

$$h_{12} = h_{21} d\frac{n_2 P_2 D_2}{n_1 P_1 D_1}.$$

A.5. GLOSSARY OF VARIABLES

Table A3

VARIABLE DEFINITIONS

VARIABLE	DEFINITION
d	Discount factor between stages
n_t	Length of stage *t*
q_i^t	Annual generation output in stage *t* of source *i*
x	Coal-fired generation
oil	Oil-fired generation
ng	Natural gas–fired generation
nu	Nuclear generation
w	Conventional renewable generation (wind, biomass, geothermal, municipal solid waste)
s	Solar generation
h2o	Hydro generation
m^i	CO_2 intensity of source *i*
E_t	Total emissions in stage *t*
$C_{it}(q_t^i)$	Cost function for generation in stage *t* of source *i* ($i = \{x, ng, nu\}$)
P_t	Consumer price of electricity in stage *t*
t_t	Price of emissions in stage *t*
f_t^i	Net tax on generation in stage *t* of source *i* ($i = \{x, ng, nu\}$)
p^i	Profits from source *i*
$G_t^j(K_t^j, q_t^j)$	Cost of renewable energy generation in stage *t* of source *j* ($j=\{w,s\}$)
$K_t^j(H_t^j, Q_t^j)$	Knowledge stock in stage *t* of renewable source *j*
H_t^j	R&D knowledge stock in stage *t* of renewable source *j*
Q_t^j	Cumulative learning-by-doing in stage *t* of renewable source *j*
h_1^j	Annual R&D knowledge generation in stage 1 for renewable source *j*
$R^j(h_1^j)$	Annual R&D expenditures in stage 1 for renewable source *j*
s_1^j	Subsidy for renewable energy generaiton in stage *t* for source *j*
s	R&D subsidy rate
r	Appropriation rate of returns from knowledge investments
v_t	Energy services in stage *t*
$u_t(v_t)$	Utility from energy services in stage *t*
U	Aggregate consumer net utility
y_t	Energy consumption rate in stage *t*
q_t^S	Percentage reductions in energy intensity from short-run investments in stage *t*

q_1^L	Percentage reductions in energy intensity from long-run investments in stage 1
$\bar{q}$	Exogenous innovation in energy intensity reductions
$Z_{j,t}(q_t^j)$	Cost of EE investments of type *j* in stage *t* ($j=\{S,L\}$)
$b_{j,t}$	Subsidy rate for EE investments of type *j* in stage *t* ($j=\{S,L\}$)
b	Perceived benefit valuation rate of EE investment
$D_t(P_t, y_t)$	Consumer demand for electricity in stage *t*
N_t	Exogenous demand growth factor
e	Very short run elasticity of electricity demand (rebound)
V	Government revenue
W	Economic surplus
r_t	Ratio of enewable to nonrenewable energy in an RPS
c_{it}	Slope of marginal cost curve in stage *t* for nonrenewable source *i*
g_{2t}^j	Slope of marginal cost curve in stage *t* for renewable source *j*
g_{1t}^j	Intercept (above P_t^0) of marginal cost curve in stage *t* for renewable source *j*
k_1^j	Learning knowledge parameter for renewable source *j*
k_2^j	R&D knowledge parameter for renewable source *j*
g_0^j	R&D investment cost parameter for renewable source *j*
g_1^j	R&D investment cost parameter for renewable source *j*
z_1^j	Intercept of marginal costs of EE improvement, for type *j* $(j = \{S_1, S_2, L\})$
z_2^j	Slope of marginal costs of EE improvement, for type *j* $(j = \{S_1, S_2, L\})$

Appendix B: Sensitivity Analysis

Let us call the parameterization described in the body of the paper as the "reference" parameterization; that includes the baseline calibration along with $r = 0.5$ and $b = 0.9$. Note that as we vary certain parameters, we continue to calibrate the model to replicate the same baseline prices and generation quantities. In this online Appendix, we consider the influence of different assumptions on policy levels and on the optimal technology policy portfolio: that is, what should be the scale of public spending on learning and R&D, in relation both to each other and to total private revenues?

B.1. SENSITIVITY OF LEARNING SUBSIDIES AND EMISSIONS PRICES

First, we focus on our optimal policies that are expressed in levels: the emissions price, subsidies for learning in wind/other, and subsidies for solar. How sensitive are our results to key assumptions, like the stringency of the emissions target or the degree of spillovers or undervaluation?

B.1.1. Stringency of emissions target

First, we consider a wider range of targets for emissions reductions. Indeed, much of the motivation for ambitious alternative energy policies in EU countries is in preparation for a transition to an energy system with dramatically less carbon. In our model, and illustrated in Figure B1, we find that a more stringent target does increase the optimal renewable subsidies (shown on the left axis); at an 80 percent reduction goal, renewable subsidies are more than double those of the 20 percent target, but those levels are still less than 1 cent/kWh for nonsolar renewables. Meanwhile, the optimal emissions price (shown on the right axis) increases by an order of magnitude, indicating that it becomes relatively more important as a policy instrument.

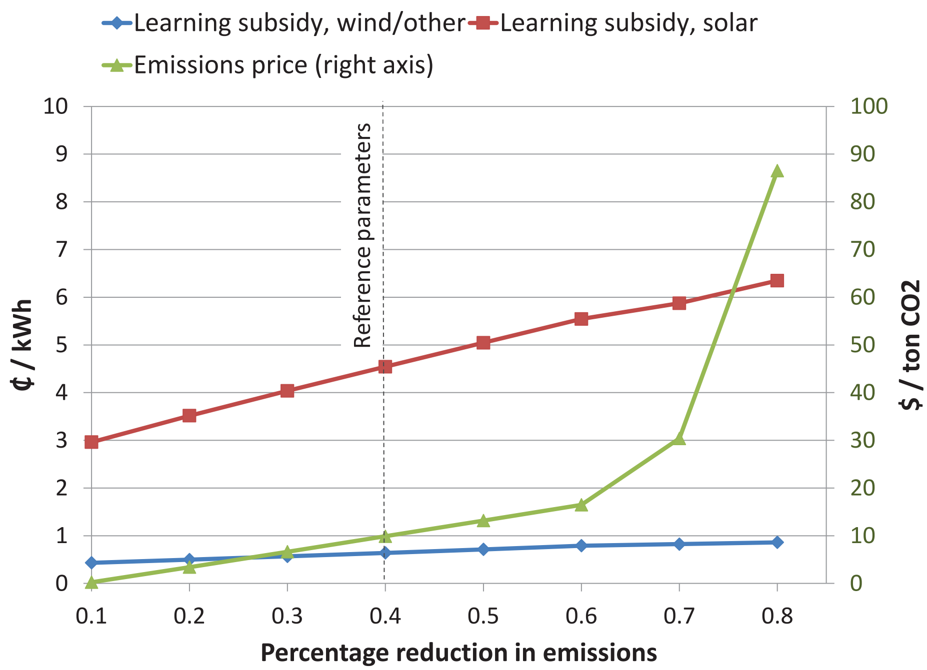

Figure B1. Sensitivity of Optimal First-Stage Policies to Emissions Target $(b = 0.9, r = 0.5)$

B.1.2. Degree of knowledge spillovers

Next, we consider the role of our market failure parameters. As modeled, the optimal R&D subsidy (s) increases one-for-one with the spillover rate $(1 - r)$. In Figure B2, we see that the optimal learning subsidy (the subsidy to renewable generation in the first stage, s_1^j) also rises proportionally with the spillover rate, with a steeper relationship for solar energy than for wind/other. Still, when we extrapolate to even higher spillover rates,[1] the optimal learning subsidy for solar energy remains under 10 cents/kWh. As larger knowledge market failures are internalized, driving larger increases renewable energy provision, the emissions price needed to meet the target falls (shown on the right axis).

[1] Baseline R&D behavior becomes unreasonable at very high spillover rates, so we limit the range of exploration.

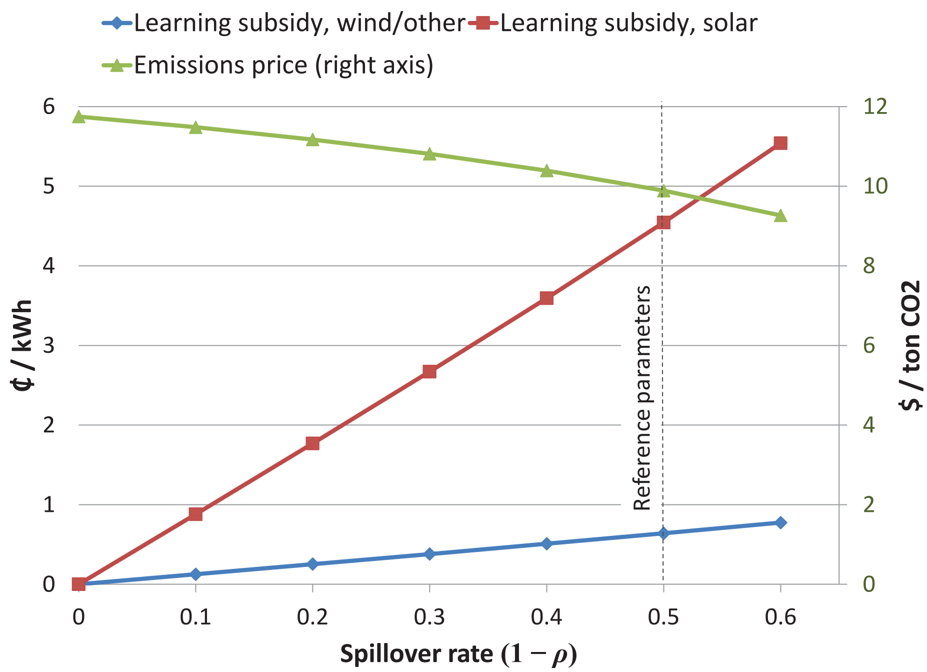

Figure B2. Sensitivity of Optimal First-Stage Policies to Knowledge Spillovers ($b = 0.9$)

B.1.3. Degree of EE undervaluation

Figure B3 illustrates the effect of higher undervaluation rates on the optimal policy mix. As energy efficiency subsidies increase to combat greater undervaluation, fewer reductions are needed elsewhere. As a consequence, both learning subsidies and the emissions price fall, and rather steeply at larger values of undervaluation.

Of course, these are optimal combinations, and it may be more difficult in practice to counteract demand-side market failures than knowledge failures. Nonetheless, in the case of uninternalized energy efficiency failures, optimal learning subsidies also fall. By driving down electricity prices, renewable subsidies exacerbate the preexisting EE market failure (Fischer et al. 2016). Thus, in either situation, greater concern about energy demand-side failures tends to undermine the case for more generous subsidies for learning through renewable energy subsidies.

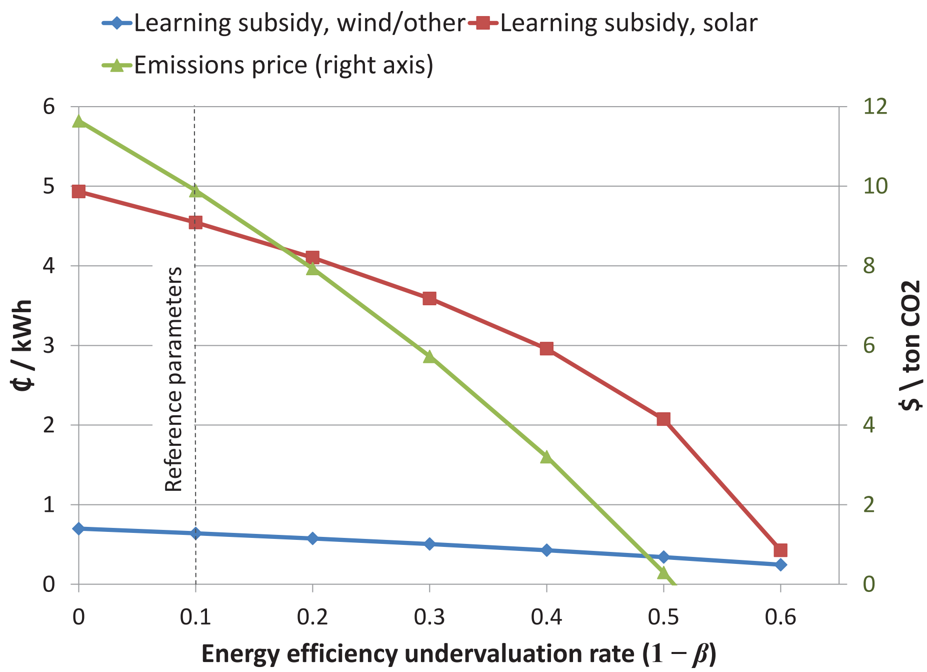

Figure B3. Sensitivity of Optimal Policies to Energy Efficiency Undervaluation ($r = 0.5$)

B.2. SENSITIVITY OF THE OPTIMAL INNOVATION PORTFOLIO

Other important assumptions are embedded in the specification of knowledge accumulation and the opportunities for cost reductions. With our reference parameterization, even with identical spillover rates for R&D and LBD, at least 80 percent of the welfare benefits of internalizing knowledge externalities come from the R&D subsidy. The reason lies in the assumed relative cost of achieving additional generation cost reductions through R&D versus LBD. For LBD, that cost is rising with the first-stage production cost curve, which is quite steep, particularly relative to the R&D investment cost curve. Although our parameters are drawn from available data, empirical evidence, and modeling practice, the true values for these specific sectors are far from certain. Thus, we construct several additional scenarios to test their relevance. Among other things, we will compute the ratio of total spending on LBD and R&D subsidies, relative to total revenues in the wind/other and solar sectors. In all scenarios, we assume there is no undervaluation of energy efficiency so that we can focus on the knowledge market failures.

The first two alternative scenarios are variations on the potential for cost reductions. First, we assume that the period for knowledge application is much longer, and we extend the second stage to 100 years ("*Long stage 2*"). With discounting, the effective weight of activities in the second stage (dn_2) increases by a third, and the benefits to knowledge spending increase accordingly, though in somewhat greater proportion for wind/other than for solar because of the larger market share for wind/other.

Second, we recognize that we may have overestimated the total cost reduction potential of second-stage generation because we assumed it applied to total generation, including previously installed capacity. In reality, innovation may bring down the supply costs not for capacity already installed in the first stage, but rather only for capacity added in the second stage. If we suppose instead that total second-stage costs equal the area under the supply curve for capacity built after the first stage ("*Lowers incremental capacity costs*"), we find that optimal learning subsidies fall roughly 20 percent for wind/other and 5 percent for solar.[2]

The next set of variations regards the knowledge production and cost functions. The third alternative scenario ("*LBD more important*") uses specifications that increase the spillovers from learning to 80 percent (while holding R&D spillovers at 50 percent), increase the cost reductions from learning ($k_1^w = 0.3, k_1^s = 0.4$), and increase the slope of R&D investment costs ($g_1 = 2$). In this case, learning subsidies contribute roughly three-quarters of the gains in economic surplus from internalizing the knowledge externality, compared with less than 20 percent in the baseline scenario.[3] In this case, the optimal learning subsidy reaches 3 cents/kWh for wind/other and nearly 9 cents/kWh for solar. Meanwhile, of total public spending on renewable energy subsidies, the portion going to deployment as opposed to R&D rises from 35 percent in the reference scenario to 87 percent for wind/other, and from 65 percent to 91 percent for solar.

However, our baseline parameters may have been more likely to err on the side of overestimating the contribution of learning to cost reductions, since few studies have attempted to separate the effects of deployment from R&D. The fourth scenario ("*Low LBD*") assumes learning is less productive ($k_1^w = 0.01, k_1^s = 0.1$), making R&D relatively more

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[2] The effects on the optimal subsidies are much smaller than the changes in second-period costs (75 percent and 50 percent lower for wind/other and solar, respectively) because the innovation parameters must be recalibrated to explain the projected R&D and learning in the no-policy baseline.

[3] Note that equilibrium cost reductions in the baseline are fixed by our calibration.

important (though not increasing k_2^w, k_2^s). This swings the optimal R&D share of total public spending to 95 percent for wind/other and just over 50 percent for solar.

Finally, lacking reasonable data on private R&D spending for renewable energy, we consider a scenario with significantly higher baseline investment, particularly for solar ("*More baseline R&D*"). Specifically, we assume baseline R&D expenditures are 5 percent for wind/other (double the reference case) and 15 percent for solar (five times the reference case).[4] The cost parameters adjust to make this spending justified in the baseline, maintaining the same degree of cost reductions. The result is more public spending on R&D in the optimum, but far less than in proportion to the baseline increase (15 percent more for wind/other and 25 percent more for solar), and only a slight complementary enhancement to LBD.

Figures A4 and A5 compare the results of these alternative sets of assumptions on the optimal supplementary technology policy portfolio. They depict total public spending on deployment (LBD) and R&D subsidies, for wind/other and solar generation, respectively, measured as a share of their total market revenues.

The results indicate that even with rather extreme parameters favoring LBD, it is difficult to drive optimal subsidies up to the 10 cents/kWh mark, even for solar. Optimal overall public spending on technological innovation seems to be in the range of 15–30 percent of market generation revenues for wind/other and 50–100 percent for solar. Meanwhile, in almost all scenarios, the ratio of deployment (LBD) spending to R&D spending does not exceed 1 for wind/other. The exception is the extreme case of "*LBD more important,*" when that ratio goes to 6.5. With our reference parameterization, solar energy is assumed to be more sensitive to R&D, but even more so to learning.

B.3. REFERENCES

Fischer, Carolyn, Michael Huebler, and Oliver Schenker. 2016. Second-best analysis of European energy policy: Is one bird in the hand worth two in the bush? University of Hanover.

Newell, Richard G. 2010. The role of markets and policies in delivering innovation for climate change mitigation. *Oxford Review of Economic Policy* 26 (2): 253–69.

[4] This percentage represents the top end of R&D expenditure shares across industries (Newell 2010).

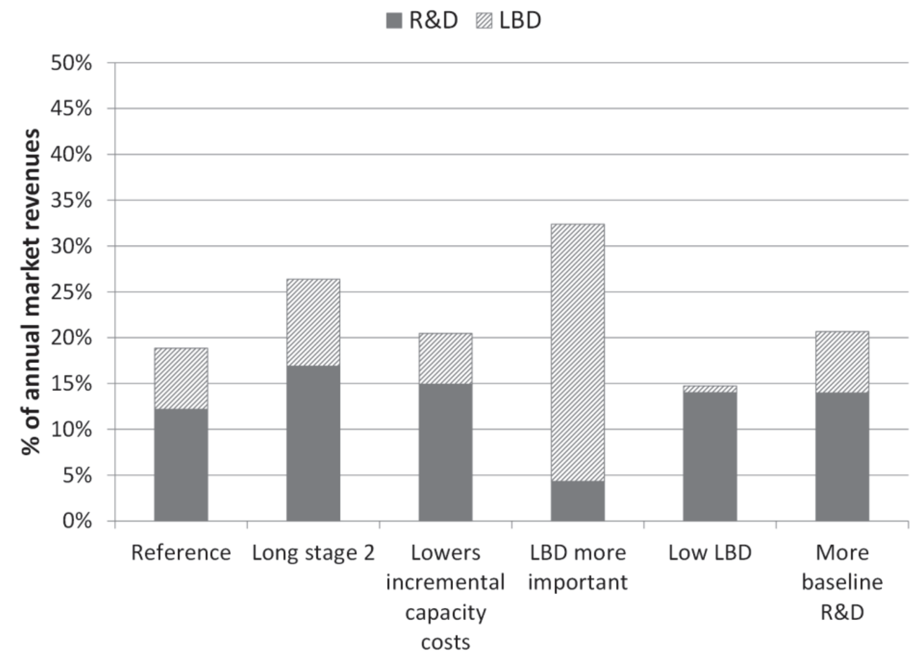

Figure B4. Optimal Public Spending on Deployment (LBD) and on R&D as Share of Total Wind/Other Revenues

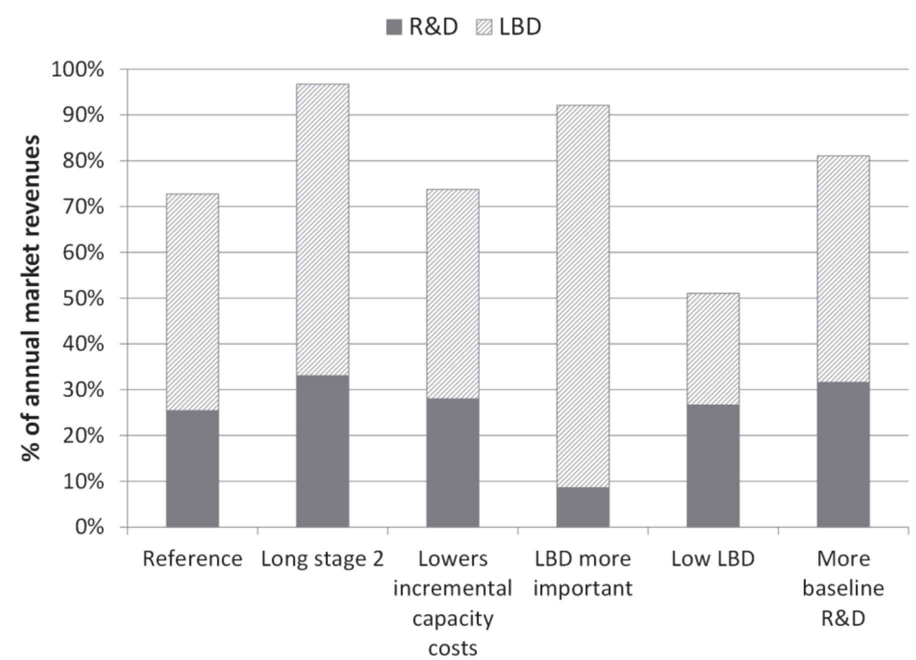

Figure B5. Optimal Public Spending on Deployment (LBD) and R&D as Share of Total Solar Revenues